Quantum Circuits 1

The Quantum Computing Stack

5	Applications	
3	Algorithms	~ CS 170
2	Circuits	~ CS 61C
4	Hardware	
1	Mechanics	~ PHYS 137 A/B

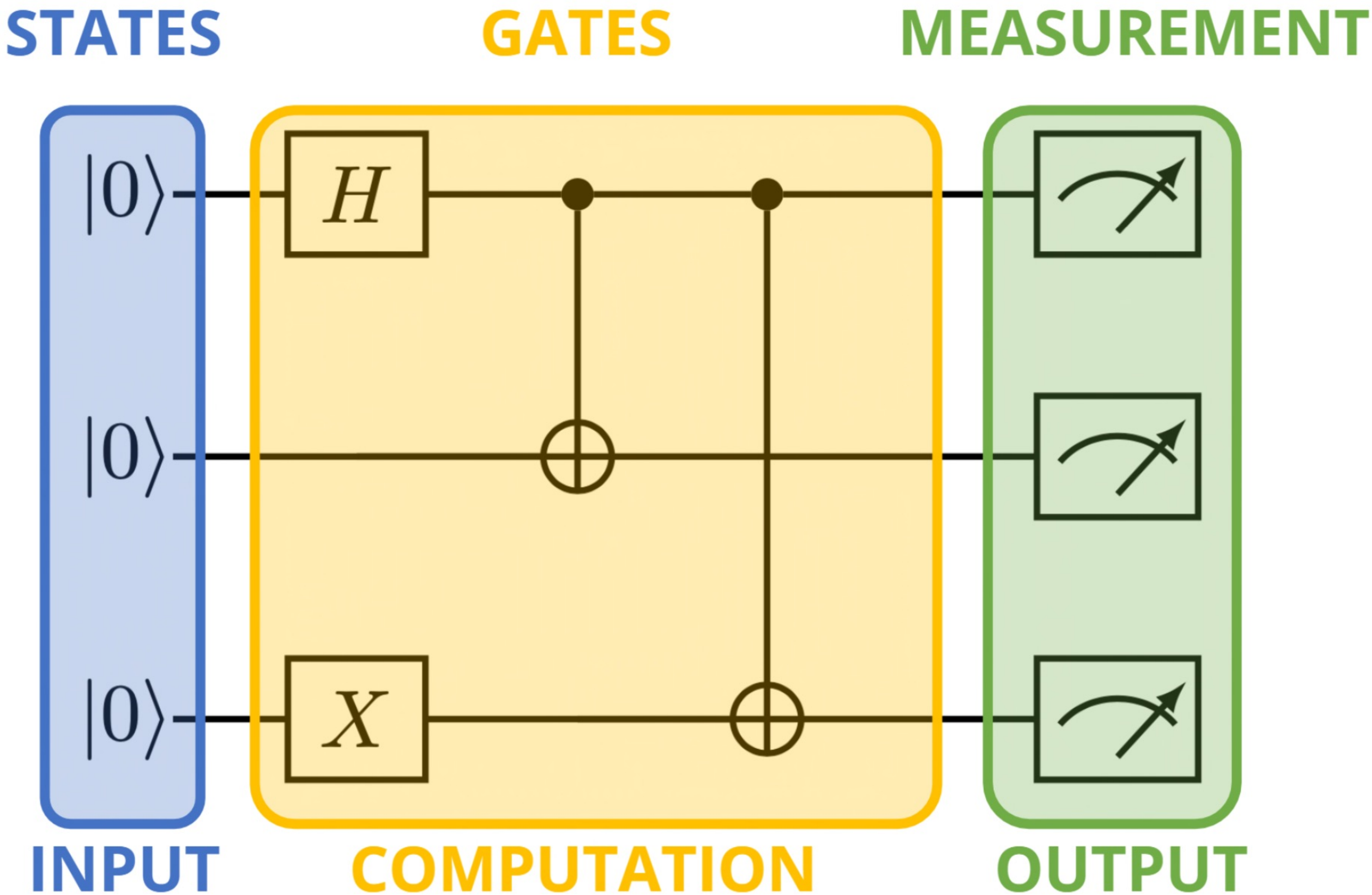
Discussion -

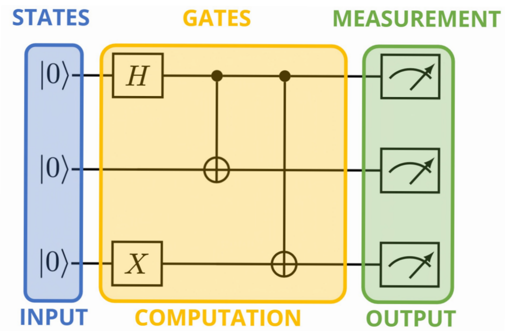
What level of the stack do you want to learn about the most? Why?

Today

- States
  - ↳ Qubit
- Gates
  - ↳ Unitary
- Measurement
  - ↳ bra-ket notation

Quantum Circuit



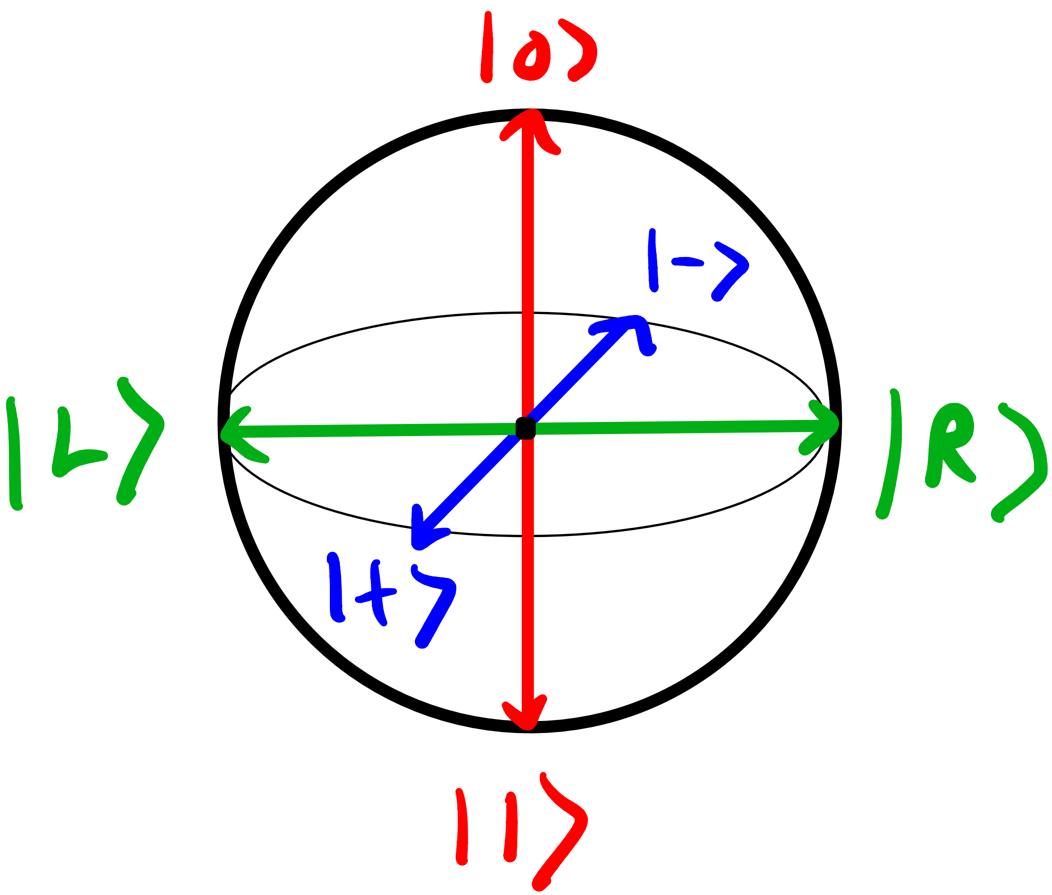


States

- Usually represented by a Ket - | >

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}\begin{matrix}0\\1\end{matrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}\begin{matrix}0\\1\end{matrix}$$

Qubit

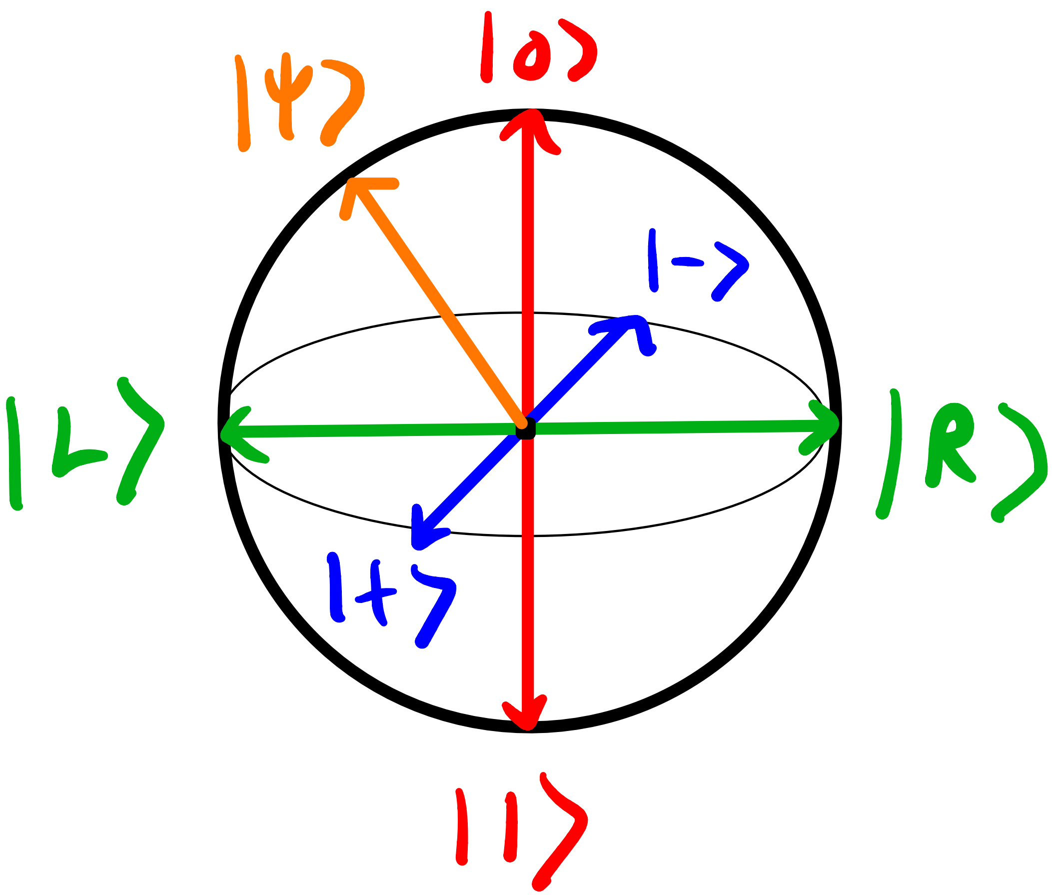


$$\frac{|0\rangle}{\begin{bmatrix}1\\0\end{bmatrix}} \perp \frac{|1\rangle}{\begin{bmatrix}0\\1\end{bmatrix}}$$

$$\frac{|+\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}} \perp \frac{|-\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}}$$

$$\frac{|R\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}} \perp \frac{|L\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}}$$

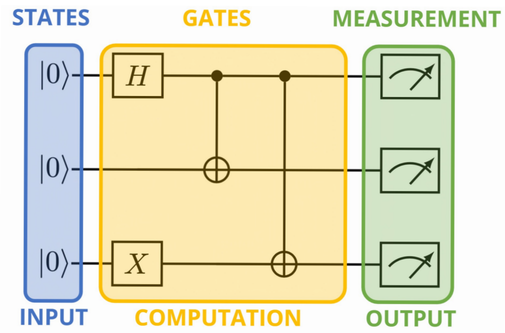
Qubit



$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\alpha, \beta \in \mathbb{C}$$



Gates

- Unitary Operators

we apply to Kets

$$\boxed{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \boxed{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

matrices!

Unitary (Why?)

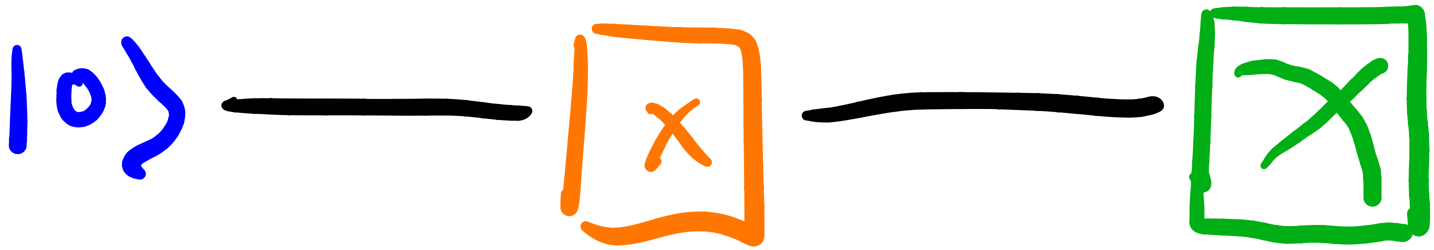
$$UU^* = UU^\dagger = I$$

$*, \dagger$ means "conjugate transpose"

i.e.: $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\quad Y^\dagger = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}^T$

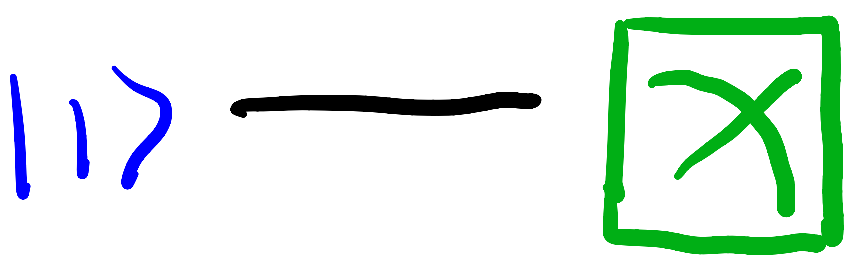
$$= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

How to "Read" a Quantum Circuit



$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

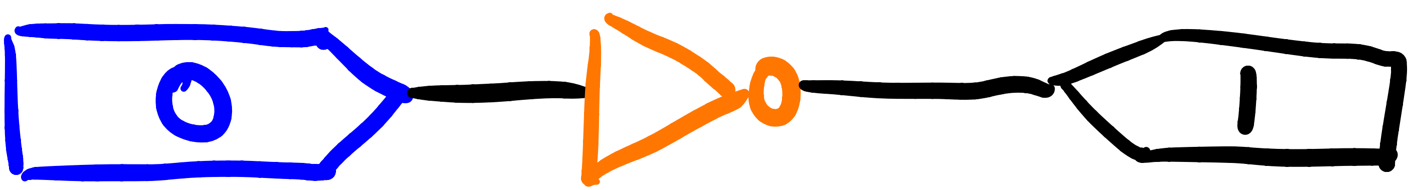


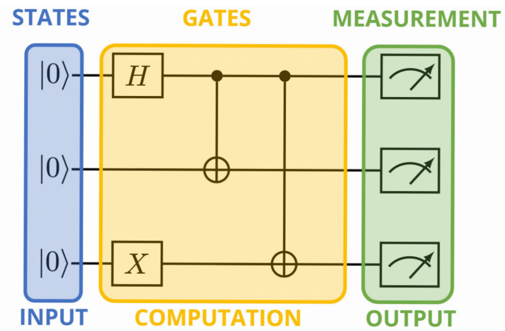
How to "Read" a Quantum Circuit

Quantum

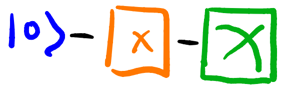
=

Classical





Measurement



- Take the magnitude of the inner product squared of our state in our computational basis $|0\rangle$, $|1\rangle$

Inner product Practice!

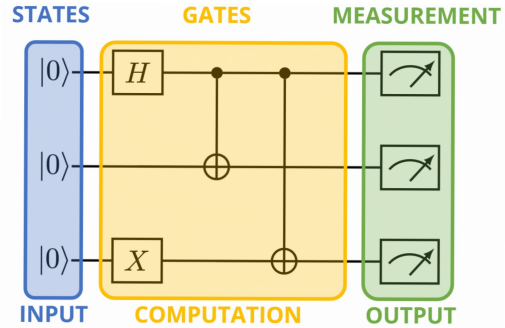
$$|i\rangle = \begin{bmatrix} 0 \\ i \end{bmatrix}$$

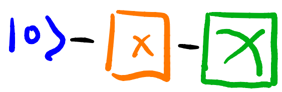
$$\langle i| = \begin{bmatrix} 0 \\ i \end{bmatrix}^{*} = \begin{bmatrix} 0 \\ i \end{bmatrix}^{\dagger} = \begin{bmatrix} 0 & -i \end{bmatrix}$$

"bra"

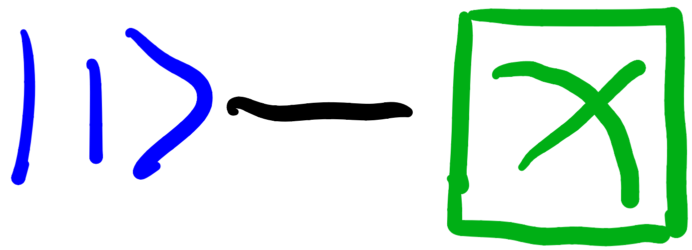
$$\langle i | i \rangle = \begin{bmatrix} 0 & -i \end{bmatrix} \begin{bmatrix} 0 \\ i \end{bmatrix} = 1$$

bra-ket





Measurement



Measuring:

$$0: \quad |\langle 0|1\rangle|^2 = 0$$

$$1: \quad |\langle 1|1\rangle|^2 = 1$$

A more Complex Circuit

$$|0\rangle \; — \; \boxed{H} \; — \; \boxed{X}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Discuss!

Keyword Chungking Express

Measuring Superposition

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = |+\rangle \quad \text{"plus ket"}$$

$0: \quad |\langle 0|+\rangle|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

$1: \quad |\langle 1|+\rangle|^2 = \left|\frac{1}{2}\right|^2 = \frac{1}{2}$

Next time!

Qubit

- How do we represent 2 or more qubits?

- Why are Quantum Gates all unitary?